

An Adaptive Multi-Operator Sequential Constructive Strategy for Routing Optimization

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Abstract

As a widely applied metaheuristic algorithm, the Genetic Algorithm (GA) performs excellently in finding near-optimal solutions. Among its components, the crossover operator plays a critical role in improving both the optimization effectiveness and the computational efficiency. The Sequential Constructive Crossover (SCX) is considered one of the most effective crossover operators; however, its greedy nature often leads to premature convergence, limiting the algorithm's global search capability. In recent years, several improvements to SCX have been proposed; Nevertheless, related research remains relatively limited. To address this issue, this paper proposes a novel Adaptive Multi-Operator Sequential Constructive Crossover strategy (AMSCX), which integrates SCX with other crossover techniques and dynamically adjusts the crossover mechanism based on the evolutionary generation. This approach enhances population diversity and improves global search performance. The experimental results demonstrate that the proposed method effectively alleviates the tendency of SCX to get trapped in local optima and significantly improves the algorithm's performance in solving the Vehicle Routing Problem (VRP).

Keywords : sequential constructive crossover, partially mapped crossover, vehicle routing problem, adaptive multi-operator strategy

1 Introduction

The Vehicle Routing Problem (VRP) [1] is a key logistics challenge. Since its introduction in 1959, it has become central in operations research. With the growing complexity of commercial operations and technological advancements, researchers have continuously sought more effective methods for optimizing vehicle routes and delivery schedules. Over time, the study of VRP has diversified into several variants—including the Capacitated Vehicle Routing Problem (CVRP), Vehicle Routing Problem with Time Windows (VRPTW), and Heterogeneous Fleet Vehicle Routing Problem (HFVRP), each addressing different operational needs and constraints. These variations reflect the complexities of real-world logistics, where factors such as delivery time windows, customer preferences, and fleet diversity must be considered. As a result, solving VRPs often requires multi-objective optimization approaches that balance cost efficiency with service quality. Despite the differences among VRP variants, their

common goal remains minimizing total delivery costs while maintaining a high level of service quality [2].

The CVRP, a core variant, deals with delivering goods under vehicle capacity constraints. The research findings [3] indicate that, to identify research trends over the past decade, a systematic review was conducted on articles published between 2010 and the first quarter of 2020. CVRP emerged as the most extensively studied variant, owing to its relatively simple structure and its ease of integration with other problem variants. Consequently, the development of algorithms based on the CVRP remains of significant importance. At the same time, with growing concerns about environmental sustainability and the efficient use of resources, researchers and industry professionals are actively exploring more advanced vehicle routing optimization algorithms to meet the evolving demands of the market and societal expectations [4].

To address the aforementioned issues, we consider the use of metaheuristic algorithms, which are renowned for their excellent global search capabilities in multi-objective problems. Various evolutionary algorithm techniques have been widely applied in practical scenarios, with some methods specifically designed to solve the VRP. For instance, the Grasshopper Optimization Algorithm [5], the Whale Optimization Algorithm [6], and the traditional Grey Wolf Optimizer [7] have all been adapted to handle different VRP variants with specific constraints. Furthermore, the Genetic Algorithm [8] has also been extended through hybrid approaches to enhance solution quality and computational efficiency.

2 Related Work

The SCX was first introduced in 2010 for solving the TSP [9] using genetic algorithms. The SCX operator constructs offspring by selecting superior edges from parent chromosomes while preserving their node sequence. It was also applied to the VRP [10]. Regarding improvements to SCX, the following related studies have been conducted. The multi-parent sequential constructive crossover (MPSCX) [11] is an improved crossover method that extends the traditional two-parent sequential constructive crossover into a multi-parent crossover, significantly enhancing the quality of the tour. A multiple-parent hybrid order and cost-based sequential constructive crossover (MPHOSCX) [12] is proposed. More genetic high-quality information is transmitted through multiple parents, while the cost-based crossover operation ensures the efficiency of the algorithm. Additionally, a nearest neighbor inverse operation is employed to enhance the algorithm's overall exploitation capability. Reverse Greedy Sequential Constructive Crossover (RGSCX) and Comprehensive Sequential Constructive Crossover (CSCX) are proposed to solve the TSP [13]. An Adaptive SCX (ASCX) [14] is introduced, which adaptively creates offspring based on the cost of the next node, allowing the selection of forward, backward, or mixed directions. The Enhanced Sequential Constructive Crossover (ESCX) [15] operator improves the performance of the SCX operator by refining the selection criteria during offspring generation. In addition to considering the actual cost of traversed cities, ESCX incorporates an estimated cost of the remaining tour, selecting the next node to construct the offspring based on this combined evaluation. The Staged Sequential Constructive Crossover (SSCX) [16] enhances population diversity by introducing new individuals, aiming to overcome the limitations of the SCX

operator.

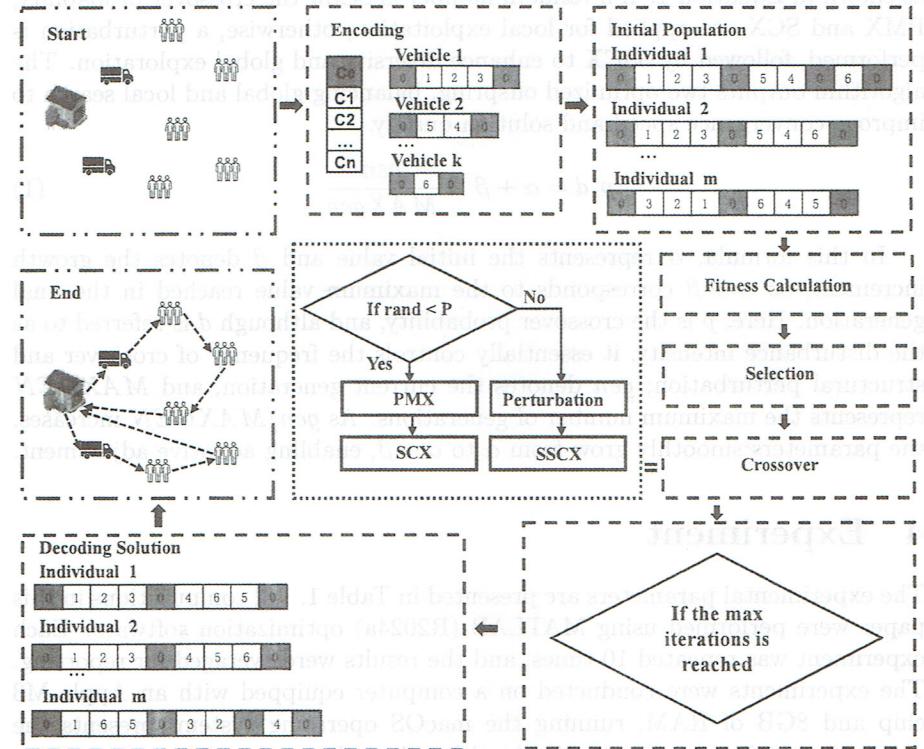


Figure 1: The flowchart of the AMSCX algorithm

3 Method

Before population initialization, individuals are encoded to meet customer demands without exceeding vehicle capacity. The algorithm then evaluates fitness and performs selection and crossover. Mutation is not included, as the focus is on crossover. After reaching the maximum iterations, the population is decoded to produce the final delivery plan shown in Figure 1.

Algorithm 1 Adaptive Multi-Operator Sequential Constructive Crossover

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1: Input:  $P_1, P_2$ 
2: Output:  $O_1, O_2$ 
3:  $p, d \leftarrow \alpha + \beta \times (gen/MAXGEN)$ 
4: if  $rand < p$  then
5:    $O_1, O_2 \leftarrow PMX$ 
6:    $O_1, O_2 \leftarrow SCX$ 
7: else
8:    $O_1, O_2 \leftarrow Disturb(d)$ 
9:    $O_1, O_2 \leftarrow SSCX$ 
10: end if
11: return  $O_1, O_2$ 

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The algorithm takes two parent individuals and dynamically adjusts the

crossover probability and disturbance intensity based on the current generation, as shown in Equation 1. If a random number is below the crossover probability, PMX and SCX are applied for local exploitation; otherwise, a perturbation is performed, followed by SSCX to enhance diversity and global exploration. The algorithm outputs two optimized offspring, balancing global and local search to improve convergence speed and solution quality.

$$p, d = \alpha + \beta \times \frac{gen}{MAXgen} \quad (1)$$

In this formula, α represents the initial value and β denotes the growth increment, so $\alpha + \beta$ corresponds to the maximum value reached in the final generation. Here, p is the crossover probability, and although d is referred to as the disturbance intensity, it essentially controls the frequency of crossover and structural perturbation; gen denotes the current generation, and $MAXGEN$ represents the maximum number of generations. As $gen/MAXGEN$ increases, the parameters smoothly grow from α to $\alpha + \beta$, enabling adaptive adjustment.

4 Experiment

The experimental parameters are presented in Table 1. All computations in this paper were performed using MATLAB (R2024a) optimization software. Each experiment was repeated 10 times, and the results were averaged for reporting. The experiments were conducted on a computer equipped with an Apple M3 chip and 8GB of RAM, running the macOS operating system, presents the parameter settings of the genetic algorithm, including a population size of 200, 1000 generations, a crossover probability (P_c) of 1.0, a mutation probability (P_m) of 0.0, parameters $\alpha = 0.3$ and $\beta = 0.2$, and a generation gap of 0.9.

Table 1: Solution Comparison Between Instances

Problem	Optimal	SCX [9]	SSCX [16]	ESCX [15]	AMSCX
E-n22-k4	375	433.7607	398.8773	407.6569	382.1334
E-n23-k3	569	686.9838	608.0915	588.1282	582.6155
E-n30-k3	534	616.5296	578.9348	577.3538	559.6946
E-n33-k4	835	931.6529	884.5905	895.0260	873.4746
E-n51-k5	521	761.9220	658.2788	632.9688	610.6101
E-n76-k7	682	1.0467e+03	841.9857	885.1311	863.8985
E-n101-k14	1067	1.5587e+03	1.3695e+03	1.4027e+03	1.3975e+03
A-n32-k5	784	874.3511	872.6946	862.8645	825.3016
A-n33-k5	661	711.5105	716.0976	709.8535	705.2793
A-n80-k10	1763	2.1066e+03	2.0027e+03	2.0553e+03	2.0274e+03
B-n31-k5	672	683.8104	683.4488	684.6724	680.0111
B-n50-k7	741	816.9760	852.3081	823.7860	804.8544

5 Conclusion

Under identical parameters, four crossover algorithms were tested. Figure 2 shows the iterative convergence of 12 instances, with summarized results in Table 1. AMSCX outperforms the others in 9 instances, performing strongly in small- to medium-scale problems and moderately in large-scale ones. Boxplots in Figure 3 confirm its stability, with lower variance and fewer outliers. The 3D iterative plot shows that AMSCX maintains high fitness, indicating superior

convergence and stability, while retaining SCX's fast iteration advantage. However, under strict constraints, AMSCX can produce infeasible solutions. Future work will aim to expand the search space and improve solution feasibility.

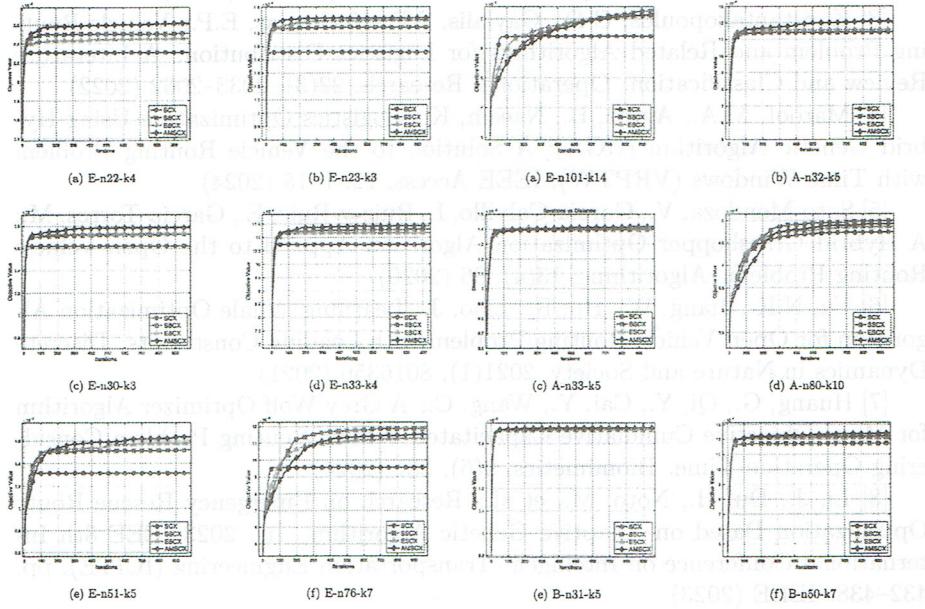


Figure 2: Results of Crossover on Different CVRP Instances, Six Subfigures on the Left and Six on the Right

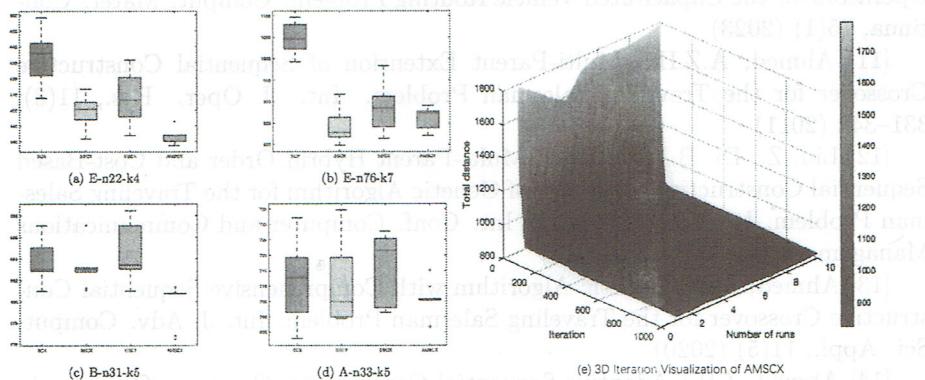


Figure 3: Box plot and 3D Iteration Visualization of AMSCX

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