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A Recursive Test for Judging M-Matrices and H-Matrices

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Abstract

A $n \times n$ complex matrix A is called H-matrix if its comparison matrix is a M-matrix. H-matrix and M-matrix are two important classes of special matrices which often appear in fields of system control and scientific computation. Many application problems are needed to judge whether a known complex matrix A is a H-matrix or not (Or equivalently, judge whether its comparison matrix $M(A)$ is a M-matrix or not). Some iterative methods and direct methods have been presented in previous researches separately. But it is difficult to predict necessary number of iterations before ending these iterative methods. And the previous almost direct methods are based on some sufficient conditions of H-matrix (For example, subset of generalized diagonally dominant matrices), so they are not always valid for all H-matrices or non H-matrices. This paper proposes a recursive direct algorithm for judging whether a $n \times n$ complex matrix is H-matrix or not. The proposed algorithm is always valid for all complex matrices (dense or sparse). The number of arithmetic operations for the proposed algorithm is one-third of some traditional algorithms like the Sweeping method, LU decomposition and the Forward backward substitution method. We also discuss a special case for the banded matrices. The proposed algorithm has a simple structure and is easy for implementation.

Keywords : H-matrix, M-matrix, Recursive Test, Computational Complexity

1 Introduction

A $n \times n$ complex matrix $A \in C^{n \times n}$ is called H-matrix if its comparison matrix is a M-matrix. H-matrix and M-matrix are two important classes of special matrices which often appear in fields of system control and scientific computation [1]. Many application problems are needed to judge whether a known complex matrix A is a H-matrix or not (Or equivalently, judge whether its comparison matrix $M(A)$ is a M-matrix or not). Some iterative methods [2-9] and direct methods [10-20] have been presented in previous researches separately. But it is difficult to predict necessary number of iterations before ending these iterative methods. And the previous almost direct methods are based on some sufficient conditions of H-matrices (For example, subset of generalized diagonally dominant matrices), so they are not always valid for all H-matrices or non H-matrices. This paper proposes a recursive direct algorithm for judging whether a $n \times n$