INFORMATION
Volume 27, Number 4, pp.221-230

ISSN 1343-4500 eISSN 1344-8994
©2024 International Information Institute

The Smallest Computational Complexity for the Linear Complementarity Problem with P-Matrix

Lei Li
Faculty of Science and Engineering, Hosei University, Koganei, Tokyo 184-8584, Japan
lilei@hosei.ac.jp

Abstract

The linear complementarity problem $LCP(A, q)$ which consists of finding an real vector $z \in R^n$ such that

$$Az + q \geq 0,$$

$$z \geq 0,$$

$$z^T(Az + q) = 0,$$

where $A \in R^{n\times n}$ and $q \in R^n$ are given real matrix and real vector respectively. When A is a P-matrix, $LCP(A, q)$ is a NP-complete problem. Some traditional direct methods for solving the above problem need $O(2^n n^2)$ or $O(2^n n^3)$ number of arithmetic operations at least. This paper proposes a recursive algorithm for solving the $LCP(A, q)$ with A is a P-matrix. The maximum computational complexity for proposed algorithm is not more than $0(2^n)$ which is the possible smallest computational complexity. And the average computational complexity for proposed algorithm is not more than $O((\frac{3}{2})^n)$. The proposed algorithm has simple structure and can be implemented easily.

Keywords : Linear complementarity problem, Recursive algorithm, P-matrix, Computational complexity.

1 Introduction

We know that the following Linear Complementarity Problem [1] often appears in fields of the mathematical programming.

$LCP(A, q)$: Let $A \in R^{n\times n}$ and $q \in R^n$, finding one or all real vectors z with satisfying

$$\begin{aligned} Az + q &\geq 0, \\ z &\geq 0, \\ z^T(Az + q) &= 0. \end{aligned} \tag{1}$$

The practical importance of the LCP is evidenced by the fact that linear programming problems, convex quadratic programming problems, and problems

DOI: https://doi.org/10.47880/inf2704-01